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WHAT MATHEMATICAL SUBJECTS SHOULD BE INTRODUCED IN THE CURRICULUM OF THE SECONDARY SCHOOL?*

BY WILLIAM BETZ.

Introduction.

The discussion of the question before us may be undertaken in two different ways. One may regard the high school curriculum as relatively fixed by traditions and external regulations over which the individual teacher or group of teachers has little or no control. In that case we should merely have to consider the merits of the various existing syllabi. Thus, in algebra we might examine the syllabus prepared by a committee of this association (published in *School Science and Mathematics,* December, 1909). In geometry we now have the National Geometry Syllabus. Other material of this sort is represented by the syllabi of such examining bodies as the College Entrance Examination Board and the Regents of the State of New York. Many good suggestions may also be found in the reports prepared by the numerous subcommittees of the International Commission.†

It seems to me, however, that in spite of the excellence of much of this available material, the very formulation of the question before us implies a different treatment. As a matter of fact, any student of secondary education knows that its most prominent characteristic just at present is a *fluctuation* of aims and values. He also knows that any attempt to solve the problems of one department of instruction without keeping in mind the educational situation as a whole is bound to fail. From this point of view all existing syllabi, even the best, are merely provisional.

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* This paper was read at the Philadelphia meeting of the Association, November, 1912. Since its original presentation, a number of changes were made in its form, in order to adapt it to a wider circle of readers. For the same reason a brief list of references was added.

† Copies of the National Geometry Syllabus and of the reports of the International Commission on the Teaching of Mathematics (American branch) may be obtained gratis by applying to the Commissioner of Education, Department of the Interior, Washington, D. C. Attention may also be called to the Syllabus of Mathematics, prepared under the auspices of the Society for the Promotion of Engineering Education, Ithaca, N. Y., 1912.

Moreover, every intelligent teacher will demand satisfactory criteria by which he may judge for himself the value of proposed plans of reorganization. Such criteria can be derived only after a careful study of the whole educational field.

If this is true, my answer to the above question would be meaningless without a brief analysis, however imperfect, of those modern educational tendencies and views which seem to have a controlling influence on the curricula of our secondary schools. In the light of this analysis I shall then try to state the principal aims of mathematical instruction. On the basis of this double inquiry it will be possible to derive a few guiding principles for the organization of the mathematical curriculum.

THE DRIFT IN SECONDARY EDUCATION.

The one dominant impression which a study of secondary education conveys is that of a rapidly shifting background. In a recent number of *Science* (November 1, 1912) Dr. W. J. Fisher, of Ithaca, N. Y., published the result of a statistical investigation of the "drift in secondary education." His figures are based on the reports of the U. S. Bureau of Education. The period covered extends from 1890 to 1910. Dr. Fisher arrives at conclusions such as the following:

1. "While the population of the continental United States has increased 50 per cent., the proportion of the population in the secondary schools has been multiplied by about *three*.

2. "While the proportion of students completing the secondary course and graduating has slightly but decidedly increased, the proportion of them preparing for college, either classical or scientific courses, has been diminished by about 60 per cent." (In 1890, 1900, and 1910, the percentages of secondary students preparing for college were 18.66, 14.53, and 6.8 respectively.)

3. "The tabular and graphical representations of statistical facts show at a glance that since 1890 the problem of the secondary school has changed from that of the fitting school to one of a decidedly non-fitting school—some bigots would say a decid-

edly *unfitting* school; a school in which only 6.8 per cent. of the pupils anticipate college work of any sort. This being the case, the colleges and universities can not lead the way in the fashion of 1892 and the Committee of Ten; the problems of secondary education can be solved only in the schools."*

The first great fact to remember, then, is that a beginning has been made in the emancipation of the high schools from college domination. What stand shall an open-minded teacher take in this matter? Certainly the demand for high school autonomy cannot be lightly set aside in the face of the above figures.

The criterion of readjustment for a given subject would seem to be that just as soon as a considerable number of efficient teachers reach the conclusion that the college requirements in that subject are out of harmony with the best interests of the majority of the pupils pursuing that subject, the high school should travel its own road irrespective of the college. Such declarations of independence have already been issued by teachers of English, of foreign languages, history, science, and other high school subjects.

Unfortunately, neither of the parties to this quarrel has avoided misrepresentation. We should not forget that we owe to the colleges a great debt of gratitude for furnishing to our secondary schools the first reliable standards of attainment at a time when the absence of such standards would have meant chaos. I believe also that the colleges are very desirous of helping the secondary schools, and that we shall continue to look to the colleges for intellectual guidance. It should not be forgotten that, in general, the American standard in secondary education is still considerably below the European standard. It is an open secret that our present policy of extending every educational opportunity to the masses has brought about a further lowering of standards. This may be unavoidable, but it does not make the fact less serious.

Obviously, the colleges cannot do their work without adequately prepared students. Besides, it is by no means certain that the best educational policy of a country consists in ignoring or neglecting the training of even the few who are destined to be the intellectual leaders. At the same time, the colleges have sinned grievously through their ignorance of modern high school conditions, through emphasizing quantity and technicalities rather than insight and quality, and through failing to supply the secondary schools with teachers having the necessary breadth of training and the vision to cope with the new situation. Such furious attacks as that made by Mr. Munroe may impress one as a little ungenerous, but they will probably go a long way toward accomplishing the necessary reforms.*

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* These inferences, like all conclusions based on statistical evidence, must be taken with a grain of salt. Some schools even now are almost exclusively college preparatory, and perhaps justly so, in view of the large number of students entering college from these schools. Each community must study its own educational needs on the basis of local statistics.

"New Demands in Education."

Most of the "new demands in education" may be grouped around three distinct ideals. These are the ideals of individualism, of vocational training, and of social efficiency. In reality they are but three aspects of the one underlying tendency away from uniformity by emphasizing personality.

Individualism.

Individualism is the characteristic feature of modern times. The student of history could easily trace its development, first in the intellectual field, as shown by the renaissance movement, then in the religious field, during the great Reformation movement of the sixteenth century, then in the political field, as indicated by the great political upheavals of the last three centuries, and finally in the industrial field, as is evidenced by the present struggle for economic freedom. In its extreme form this tendency leads to egoism and to the questioning of all restraints of personal freedom. One writer gives it the beautiful Greek name "eleutheromania," which he defines as "the instinct to throw off not simply outer and artificial limitations, but all limitations whatsoever." He goes on to say that for a century the world has been fed on a steady diet of revolt.†

The school, we are told by the individualist, must treat each child according to his own native capacities. All his latent pos-

* See J. Ph. Munroe, "New Demands in Education," Doubleday, Page and Co., 1912. Chapter 13: "How the Colleges Ruin the High Schools."

† Irving Babbitt, "The New Laocoon," 1910.

sibilities of body, mind and soul must be conserved and developed, and not dwarfed.

VOCATIONAL TRAINING.

This individualized pupil, so we are informed, must be prepared for life.

But for what sort of life? Since college preparation is to be largely ruled out, the answer commonly given is: for social, commercial, and industrial life. Preparation for a vocation is the watchword of many an ardent reformer. But *which* vocation is to be recommended to the individual pupil?

The difficulties we encounter at this point are enormous. Thus far, a satisfactory scheme for industrial education is yet to be announced. What may be done and perhaps should be done can be inferred from such sources as Leavitt's Examples of Industrial Education, Munroe's New Demands in Education, and the bulletins of the National Society for the Promotion of Industrial Education.*

EFFICIENCY.

The problem of vocational training will eventually be solved, but perhaps not in our day, if only for financial reasons. Meanwhile, we are assured that the new high school will be subjected to a rigid test of *efficiency*.

This demand should be welcomed enthusiastically by all teachers. However, in many of the current efficiency arguments there is an element of charlatanism, of gross materialism, and of loose thinking. Some efficiency experts would have us think that the work of a school can be gauged like the output of a factory. It should not be forgotten that the finest and most lasting results of good teaching cannot be tested by the methods of the counting-room or measured in dollars and cents, and in fractions of a second. It is the very effort to examine, examine, examine, that has brought the school into such a formalistic rut. Let the examination enthusiasts read what Dr. J. J. Finlay of Manchester University has to say of our essentially British examination system.*

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* See also the bulletins of the National Association of Manufacturers of the United States of America (170 Broadway, New York); the very extensive report of the N. E. A. committees, printed in the 1910 volume of the Proceedings of the National Education Association; the bulletins of the U. S. Bureau of Education; Report on Vocational Training in Chicago, City Club of Chicago, 1912; G. Kerschensteiner, "Education for Citizenship," Rand, McNally & Co., 1911 (prize essay); R. M. Weeks, "The People's School," Houghton Mifflin Co., 1912; Bloomfield, "Vocational Guidance of Youth," and Snedden, "The Problem of Vocational Education," Houghton Mifflin Co., 1911 and 1910 respectively.

It cannot be denied that there is a semi-humorous inconsistency in the attitude of some of our most ardent school reformers. On the one hand, the school course is to be adapted to the individual. The child is to study only the things that appeal to him. If the English classics are uninteresting, let him read magazines. If algebra or Latin are too hard, let them be dropped. On the other hand, efficiency means the living up to definite standards, which is the very opposite of this perpetual coddling and fondling. A common-sense middle ground is necessary. As Munroe puts it: "Every high school boy is a problem by himself; and the business of the high school is to develop him, as an individual, to his highest possible usefulness as a man and as a citizen. To be so developed, he must, in the first place, be disciplined by hard, serious, steady work; to do that hard work he must be interested in it; and, to be interested in it, he must himself see that it is going to be of use" (p. 210, *l. c.*).

ATTEMPTS IN REORGANIZATION.

By what means may the high school hope to meet these demands? The answer is threefold.

First, by a thorough study of current criticisms of its present system, followed by a restatement of educational aims and values in the light of past and present educational history. In this way alone can we expect to gain the necessary insight to meet the problem of individual development.†

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* "The School," Home University Library, Holt & Co., 1912, pp. 124–131.

† In Johnston's High School Education, for example, 21 serious criticisms against our present system are enumerated. Only one of these can be mentioned here, namely the statement that "enormously large numbers withdraw from school."

What are the facts?

According to a recent report of the U. S. Bureau of Education the number of students enrolled in *public* high schools in 1911 was 984,677. Of these, 421,335 were in the first year of the high school.

Second, by extending the opportunity to acquire some form of secondary education to "all the children of all the people." This will meet the problems of vocational training.

Third, by adopting the six-year curriculum. This is our principal remedy in the direction of greater efficiency.

A discussion of these essential principles of reorganization is impossible at this point. I can only refer to such books as

Sachs, "The American Secondary School," The MacMillan Co., 1912;
Munroe, "New Demands in Education" (see above);
Johnston, "High School Education," Scribners, 1912;
Bagley, "Educational Values," The MacMillan Co., 1911;
Hanus, "A Modern School," The MacMillan Co., 1909;
Hanus, "Educational Aims and Educational Values," The MacMillan Co., 1902.

Concerning a restatement of aims and values, it may safely be asserted that no teacher can expect to do justice to his subject without studying carefully the modern point of view in regard to such questions as the theory of mental discipline. Rüdiger's summary of recent educational theory is very helpful.* Contrary to the usual impression conveyed by superficial alarmists it may be said confidently that "we have neither proved nor disproved the formal discipline dogma" (Johnston, p. 37). It is very probable that the painstaking investigations of psychologists will strengthen the position of mathematics in the curriculum, always with the proviso that a teacher knows what he is really doing. I even believe that we are on the threshold of a return to drill methods.†

"The age census for 1910 is not yet published, but estimates based upon the percentages for 1900 indicate that the number of children 5 to 18 was approximately 25,000,000 in 1910. About 1,824,000 of these, or 7.28 per cent., were in the 14 year group. The 13, 15, and 16 year groups are very close to this number. We should expect this 1,824,000 group to furnish the students for the first year of the high school. We find 421,335, or 23.1, in the first year of the high school in 1911." (Quoted from a letter of Mr. A. Summers, statistician of the Bureau of Education.) These figures tell their own story.

* W. C. Rüdiger, "The Principles of Education," Houghton Mifflin Co. 1910, Chap. VI.

† How does this passage impress the timid teacher who had gained the idea that all drill was forever tabooed:

As to the extension of high school facilities to the masses, thus making the high school a real "people's college," it may be said that at present a conflict is going on between the cosmopolitan type of high school and the specialized high school. Each has its merits and its defects. The cosmopolitan type is better as a selective agency, since it makes a wider appeal to individual aptitudes, while the specialized high school will undoubtedly do better work in its own field.

Finally, the adoption of a six-year curriculum is only a question of time, whichever of the four possible forms it takes. Recent popular and educational discussion is enthusiastically in favor of this reform. Personal inquiry of a score of partial or complete adoptions of the plan in the United States brings me the conviction that it is meeting with all the success which could be expected under the circumstances.*

AIMS OF MATHEMATICAL TEACHING.

Having now completed a hasty examination of the educational background, we may proceed with more assurance to a consideration of our main question, provided we can agree on the aims of mathematical teaching in our secondary schools.

"The cry for high school freedom from domination, even when justified, has carried with it the tone of impatience in directing mental processes all of which must be slowly, painstakingly, and expertly supervised if they are to be permanently worth while. Insolent and careless intellectual attitudes, superficiality and lack of thoroughness or of finished knowledge of a few fundamental subjects, inability and disinclination to think an issue or a problem through to its minute details, halfway racings into utilitarian fields, a tendency to tackle anything but the trunk of the curricula, novelty specifics in which pedagogic mastery of the new material is not assured, all characterize questionable traits in high school graduates and suggest that we must regain our faith in the ultimate values and permanent results of drilled training such as some of the older disciplines and traditional models seem still uniquely to afford." (Ch. H. Johnston, "High School Education," Chap. 2, p. 31.)

* See J. Sachs, "The American Secondary School," p. 108, The Macmillan Co., 1912; Ch. H. Johnston, "High School Education," Scribners, 1912, Chap. IV.; Bulletin, 1911, No. 16, whole number 463, United States Bureau of Education, containing a report on the six-year curriculum from the standpoint of mathematics; *The School Review,* December, 1912 (pp. 665-688), January, 1913 (pp. 1-25); *The Ladies' Home Journal,* February, 1913, March, 1913, April, 1913.

Mr. T. Brookman, in his splendid article on first-year high school mathematics,* says with only too much truth: "It is difficult to find an instructor in mathematics who has thought out the problems of first-year algebra and why it *is* taught or why it *should* be taught to all students. He only knows that he is thankful that the subject does not fall to his lot." Does not this ignorance account for many of our troubles?

The aims of mathematical teaching center around the two ideas of insight and efficiency, which may also be described by the terms impression and expression, or by form and content. The one is subjective, the other objective. With this main classification in mind, we may distinguish these five important aims of mathematical teaching:

1. *Symbolism.*—The pupil must be made to appreciate the value of mathematical symbolism as man's chief tool for summarizing important quantitative relations. He should respect a *formula* as a condensed record of human effort and achievement. Mathematics furnishes a language without which, it is not too much to say, our present civilization cannot be understood and cannot exist.

2. *Applications.*—A living language like mathematics exists for the expression of ideas. Hence, the pupil should be impressed with the reality of mathematics by numerous applications lying within his grasp.

3. *Function Concept.*—The function concept, together with the equation, should be given a prominent place. Training in functional thinking is almost a necessity for the modern mind. It is at the bottom of the doctrine of evolution. Without it modern science remains a jumble. More than that, the function concept furnishes a unifying bond for all branches of mathematics.

4. *Space Intuition.*—There should be constant insistence on a cultivation of space intuition. The training obtained in this way is not only extremely valuable for mathematical purposes, but is also indispensable to the mechanic, the designer, the representative artist.

5. *Logic.*—The pupil should learn to understand and to appreciate the logic of mathematics, and he should show evidence of

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* *Teachers College Record,* March, 1909.

this training in all his other work. As a major aim this is second to no other, a fact which has been recognized since the days of Plato. The constant necessity of checking results and of giving reasons for statements is conducive to accuracy in an incomparable way. Besides, the habitual use of inductive and deductive reasoning in mathematical teaching familiarizes the student with the two principal modes of discovering and of testing intellectual truths.

It is very doubtful whether any other high school subject can produce equally cogent and permanent reasons for being included in the curriculum. This becomes all the more apparent if it is observed that each of these five aims is essential from either a purely ideal or a narrowly utilitarian standpoint. To borrow a happy phrase of Professor D. E. Smith, mathematics is to a remarkable degree "potentially practical."* It may be said without hesitation that if the opponents of mathematics should ever succeed in abolishing it from the curriculum, public opinion would demand its restoration within a year. Mathematics meets a fundamental human need.† Finally, let it be remembered that ours is an age of mental unrest and uncertainty. To acquaint the minds of the young at this critical period with reliable standards of intellectual certainty is a service no less real because it is often overlooked, a service that endows mathematics with the highest moral value.

The Mathematical Curriculum.

Unfortunately there is very little doubt that the mathematical curriculum at present is not doing justice to the five aims enumerated above. The chief hindrances preventing their more perfect realization are the compartment system of teaching, the four-year curriculum, and the poor preparation of pupils and teachers.*

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* *Bulletin of the American Mathematical Society,* February, 1913, p. 250.

† For a discussion of the old conflict between realism and idealism in mathematical teaching see D. E. Smith, "The Teaching of Geometry," Ginn & Co., 1911, pp. 7–25; Th. J. McCormack, "Why Do We Study Mathematics: a Philosophical and Historical Retrospect," Cedar Rapids, Iowa, 1910 (an abstract of this paper may also be found in the N. E. A. Proceedings of 1910); W. H. Metzler, "Educational Value of Mathematics" Journal of Pedagogy, June 1905, "Formal Discipline," This Magazine, September 1910, "Mathematics for Training and Culture," Proc. Ass. Coll. Prep. Schools Middle States and Maryland 1910. See also the preceding number of *The Mathematics Teacher* (March, 1913).

I shall begin this brief discussion of the mathematical curriculum by quoting Professor Felix Klein's definition of elementary mathematics: "In all domains of mathematics we may call those parts 'elementary' which may be understood by a person of average ability without long continued special study."† He limits this definition, however, by assigning to the school those elementary topics only which are necessary to a comprehension of our modern civilization. In the opinion of Professor Klein this comprehension can best be secured by making the function concept in geometric form the very heart of all mathematical teaching (p. 34, *l. c.*).

It will be seen that this broad definition still leaves a considerable amount of freedom to the individual teacher. This freedom is checked chiefly by the teacher's limited preparation and by the requirements of examining bodies. Besides, each country in the course of time has developed its own mathematical routine which it would be difficult or useless for the individual teacher to ignore. Reforms, accordingly, must be a matter of evolution.

With these limitations in mind, I submit the following topical outline in synoptic form.

(In arranging the algebraic topics the original intention was to show the dependence of every topic on the preceding topics in the same column and in the adjacent columns. This plan was carried out but partially. Otherwise the table is self-explanatory.)

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* See Report of International Commission, Bulletin 16, 1911, p. 94; Th. J. McCormack, "Education, Utopian and Real," *School and Home Education*, January, 1913; J. E. Russell, "Professional Factors in the Training of the High School Teacher," *Educational Review*, March, 1913.

† Klein und Schimmack, "Der Mathematische Unterricht an den Höheren Schulen," Teil 1, p. 111.

Outline of Mathematical Work that Should be Required of Every Student in a General High School Course.

Elementary Algebra.

I Number-System.

1. Positive Integers.
2. Negative Integers.
3. Fractions.
4. Irrational Numbers.
5. Imaginaries.

II Operations.

1. Addition.
2. Subtraction.
3. Multiplication.
4. Division. (H. C. F., L. C. M., Factoring.)
5. Involution.
6. Evolution.
7. Logarithmation.

III Equations.

1. Simple.
2. Fractional.
3. Simultaneous Linear.
4. Radical.
5. Quadratic.
6. Exponential.

IV Function-Concept.

1. Statistical Graphs.
2. Graphs of Formulas.
3. Graphic Solution of Problems.
4. Graphs of Linear Functions.
5. Graphic Extraction of Roots.
6. Graphic Solution of Quadratic and Higher Equations.

V Applications.

1. Arithmetic.
2. Geometry.
3. Mechanics.
4. Science.
5. Economics.
6. Statistics.
7. Shop Mathematics.
8. Slide Rule.

Geometry (Plane and Solid).

Symmetry, Congruence, Equality, Similarity. (Follow National Geometry Syllabus.)

Trigonometry (Plane).

(Follow suggestions of American Mathematical Society. See Syllabus of College Entrance Examination Board.)

In the preparation of the above table only a desirable minimum requirement was kept in mind. For a discussion of the separate topics I may refer to the report on "Mathematics in the Public and Private Secondary Schools in the United States," prepared by Committee III of the International Commission (Bulletin 16, pp. 15–75.) Hence I can bring these considerations to a close by adding the following guiding principles for the administration of the mathematical curriculum, as well as some statements that may serve to justify the extension of the above table beyond the customary requirements.

1 The mathematical curriculum of the future must meet the wants of all types of students. For some of these types only

actual experiment can determine the most desirable selection and sequence of topics.

2. Each topic should be viewed in the light of Professor Klein's definition of elementary mathematics.

3. The five aims stated above must be met at least approximately, if the cultural and the practical value of mathematics are to receive due weight.

4. These five aims cannot be fully realized unless every student in the general course is required to become familiar with the elements of algebra, of plane and solid geometry, and of plane trigonometry.*

* If any one should consider this an unreasonable demand, let him make a brief study of the European requirements. If America wishes to compete with Europe along industrial, technical, or scientific lines, it cannot ignore the increased educational opportunities given to European secondary students. In this connection the following passage may be of interest (taken from a report of Professor D. E. Smith on the educational work of the Fifth International Congress of Mathematicians, held at Cambridge, England, last August):

"For example, we have generally failed to do anything worth while with algebra and geometry in the elementary school, altho there is hardly a country among those reporting at Cambridge that does not make a success of this work. They do it by cutting out most of the subjects treated of in the last two years of our arithmetic course, and putting the classes in the hands of the same teachers of mathematics that are to present the subject in the years corresponding to our high school. This enables them to prepare the pupil's mind for a serious study of algebra, geometry, and trigonometry in the next two years, and to cover much more ground than we do. Furthermore, they are not content with stopping at quadratics and plane geometry, but it is a growing custom to add to the solid geometry and trigonometry a course in analytic geometry and the calculus in the secondary school, a custom that is pretty sure to be adopted in such of our schools as have teachers capable of understanding it and superintendents sympathetic with progress. If it is said that the calculus must be very superficial if done by pupils before entering college, it may be replied that in one large English school for boys, recently visited by the writer, about half of the calculus written by Professor Osgood was covered, the examinations being set by outside authority and the results being in every way satisfactory. Moreover, the subject was not elective but was required of every boy in the school. It is needless to say that this was exceptional, but good courses, even tho of lower grade than this, are given in many schools, and there is a growing literature upon the subject. Our courses are more systematic than those found in the Teutonic countries, resembling somewhat more

5. The function concept in geometric form should be regarded not as an additional topic, but as an organizing principle of the highest value.

6. *Solid Geometry* is necessary to secure training in space intuition, and to open the door to numerous applications. The teaching of plane geometry alone gives the pupil a false impression. We live in a world of three dimensions, and not in a "Flatland."

7. Trigonometry is necessary for its great practical value in all departments of mechanics and engineering, not to mention surveying. It also gives the pupil valuable numerical training in the use of tables, and it affords opportunity for a thorough study of logarithms and of the slide rule.

8. The topics should be so selected and arranged that the psychological element in the development of the pupil is kept in mind. To a certain extent this order coincides with the historical order. There should be a transition from the concrete to the abstract. Great attention should be given to the time element in the assimilation of new ideas. Just as arithmetic furnishes the necessary background for algebra, so we should have a preliminary course in plane geometry, in solid geometry, and in trigonometry. The experience of a century in European schools proves the correctness of this view.*

9. Any principles or topics which cannot be given adequate scientific treatment, or which have too little significance to the

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those found among the Latin peoples. For example, our geometry is more systematically arranged than that of Germany, resembling more that to be found in France or Italy. The blind attack upon all mathematics, which seems to be a phase of degeneracy in some of our educational circles today, is not found in the best European countries. On the contrary there is a general desire on the part of all to hold to the best that we have, while constructively modifying the weaker parts of the edifice." (*Educational Review,* January, 1913, p. 3.)

* See P. Treutlein, "Der Geometrische Anschauungsunterricht," Leipzig, 1911 (an excellent book, containing a splendid bibliography); G. C. Young and W. H. Young, "Der kleine Geometer," Leipzig, 1908 (a German translation of the corresponding English text by the same authors); C. A. Laisant, "Initiation mathématique," Hachette and Co., Paris, 1906; G. Veronese, "Elementi di Geometria intuitiva," Padova, 1906; D. E. Smith, "L'enseignement of Mathématique," November, 1912, pp. 507–527; W. Betz, "Intuition and Logic in Geometry," *The Mathematics Teacher,* September, 1909, p. 3.

pupil, should either be assumed without proof, or be omitted entirely. To this category belongs such material as the "incommensurable case,"* the theory of limits, cube root, complicated work in factoring, in fractions, in involution and evolution.

10. The pupil should gain a conception of the significance and the organic relation of the principal divisions of the subject. Whether this is to be done by a fusion of the different branches of mathematics or by expert correlation, must be determined ultimately by more careful and extensive experimentation.

If one prefers to begin with an unsystematic "problem course" under the general name of "mathematics," care should be taken that a system of some kind is developed in the end. Otherwise the result will invariably be confusion and inefficiency. On the other hand, the usual systematic courses should be so arranged that contact with life is not lost and that the pupil is not given a wrong impression at the very outset by useless technicalities. In general, a minimum mathematical basis with which the pupil has become perfectly familiar is vastly better than a mass of unassimilated material.

11. The course in algebra should represent the organic interweaving of these five elements: the number classes, the operations, the equation, the function concept, and the applications.

If a comparison be permitted, it may be said that the number-system of algebra furnishes the raw material, the *vocabulary*, of our mathematical language. The operations represent the transformations of declension and conjugation, thus creating flexibility of *form*.† The equation is the *sentence* of algebra. It is the chief tool for the statement and solution of problems. But the function is an entire paragraph, the development of an idea. The equation is only an incident in a functional development. The function is dynamic, while the equation is static.

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* For a discussion of the "incommensurable case" see the National Geometry Syllabus, p. 40; also Arthur Schultze, "The Teaching of Mathematics in Secondary Schools," The MacMillan Co., 1912, pp. 188–196.

† It would be possible to extend the linguistic analogy still further by introducing at this point another division of algebraic work which may be said to correspond to the syntax of a language. Into this division could be placed the fundamental postulates and the typical processes and rules, such as the rule for squaring $a + b$ and the rules for factoring polynomials.

And just as an essay must have a worthy theme, so this mathematical language can display its power only by effective applications. The problem work of algebra corresponds to the writing of complete essays. The teacher of English introduces different types of composition and treats a great variety of themes. The teacher of algebra will try to secure mathematical fluency by a sufficiently elastic assortment of problems.

12. The topics of geometry group themselves organically around the fundamental ideas of symmetry, of congruence, equality, and similarity. The locus problem should receive very much greater emphasis, for it represents the dynamic, functional element of geometry. Above all, opportunity should be given not merely for a reproduction of book theorems, but also for the independent discovery and proof of geometric truths.

CONCLUSION.

In short, there is no reason why the curriculum in secondary mathematics, if properly administered, should not create in a large number of pupils some of that genuine enthusiasm and interest with which mathematics—the most perfect of all sciences—has never failed to inspire the leaders in the long upward trend of human development.

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